

ISAR IMAGE RECONSTRUCTION KALMAN APPROACH TO LINEAR FREQUENCY MODULATED SUGNALS

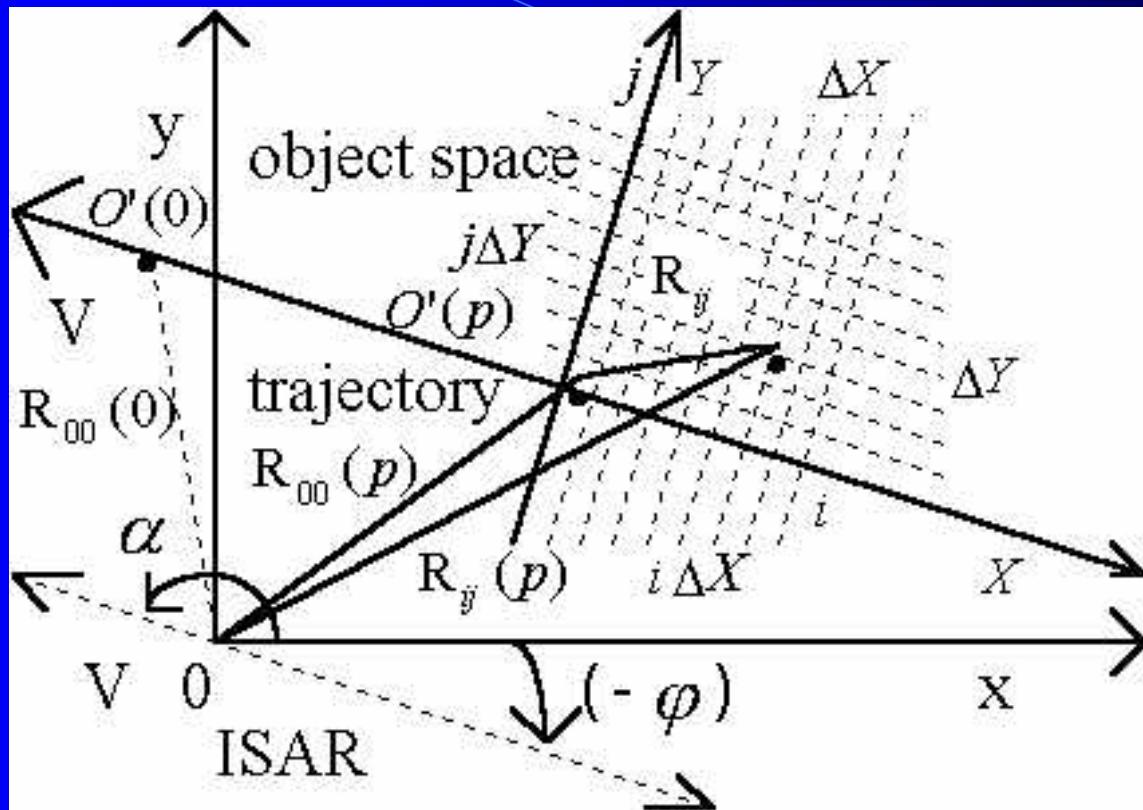
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Goals:

- Developing ISAR geometry
- Modeling ISAR signals with inner LFM
- Defining image extraction recurrent Kalman procedure and constructing approximation functions-models.
- Creating basic matrices and developing image reconstruction Kalman procedure
- Numerical experiment of Kalman object intensities estimation and geometry extraction procedure from LFM simulated ISAR data

ISAR 2-D geometry



4. Geometric vectors:

$$\mathbf{R}_{ij} = [X_{ij}, Y_{ij}]^T$$

$$Y_{ij} = j\Delta X$$

$$X_{ij} = i\Delta X$$

5. Vector velocity:

$$\mathbf{V} = [V_x, V_y]^T$$

1. ISAR range vector equation: $\mathbf{R}_{ij}(p) = \mathbf{R}_{00}(0) + \mathbf{V} \left(\frac{N}{2} - p \right) T_p + \mathbf{A} \mathbf{R}_{ij}$

2. Range vector at the moment $N/2$: $\mathbf{R}_{00}(0) = [x_{00}(0), y_{00}(0)]^T$

3. Transition matrix: $\mathbf{A} = \begin{bmatrix} \cos j & \sin j \\ -\sin j & \cos j \end{bmatrix}$

Modeling LFM ISAR Signal

1. ISAR signal return $\mathbf{x}(p, k) = S(p, k) + n(p, k)$

2. ISAR deterministic signal

$$S(p, k) = \sum_i \sum_j a_{ij} \text{rect} \frac{t - t_{ij}}{T} \exp \left\{ j \left[(t - t_{ij}) + b(t - t_{ij})^2 \right] \right\}$$

$$\text{rect} \frac{t - t_{ij}}{T} = \begin{cases} 0, & \frac{t - t_{ij}(p)}{T} \leq 0 \\ 1, & 0 < \frac{t - t_{ij}(p)}{T} \leq 1 \\ 0, & \frac{t - t_{ij}(p)}{T} > 1 \end{cases}$$

$$t = t_{ij \min}(p) + (k - 1)\Delta T$$

$$k = 1, K + L$$

$$t_{ij \min}(p) = \frac{2R_{ij \min}(p)}{c}$$

$$t_{ij \max}(p) = \frac{2R_{ij \max}(p)}{c}$$

$$L = \frac{t_{ij \max}(p) - t_{ij \min}(p)}{\Delta T}$$

3. Time dimension of the object

4. LFM rate: $b = \frac{p\Delta F}{T}$

5. Time delay of ij -th point scatterer: $t_{ij}(p) = \frac{2R_{ij}(p)}{c}$

6. Dimension of the ISAR line of sight to ij -th point scatterer:

$$R_{ij}(p) = \left[x_{ij}^2(p) + x_{ij}^2(p) + \right]^{\frac{1}{2}}$$

7. Coordinates of ij -th point scatterer of the object space:

$$x_{ij}(p) = x_{00}(0) + V_x T_p \left(\frac{N}{2} - p \right) + X_{ij} \cos j + Y_{ij} \sin j$$

$$y_{ij}(p) = y_{00}(0) + V_y T_p \left(\frac{N}{2} - p \right) - X_{ij} \sin j + Y_{ij} \cos j$$

8. Components of vector-velocity: $V_x = V \cos a$ $V_x = V \cos a$

Kalman Image Reconstruction Procedure

A. Vector Measurement Equation and State Equation:

$$\hat{\mathbf{I}}(p, k) = \mathbf{S}[p, k, \mathbf{a}(p)] + \mathbf{n}(p, k)$$

$$\mathbf{a}(p) = \mathbf{g}[p, k, \mathbf{a}(p-1)] + \mathbf{n}_0(p, k)$$

$$\mathbf{g}[p, k, \mathbf{a}(p-1)] = \mathbf{g}(p, k) \cdot \dot{\mathbf{a}}(p-1)$$

B. Approximation Functions:

$$\mathbf{S}[p, k, \mathbf{a}(p)] = \mathbf{S}_c[p, k, \mathbf{a}(p)] + j\mathbf{S}_s[p, k, \mathbf{a}(p)]$$

Taylor expansion:

$$\mathbf{S}_c[p, k, \mathbf{a}(p)] = \mathbf{S}_c[p, k, \dot{\mathbf{a}}(p)] + \sum_{j=1}^J \sum_{i=1}^I \frac{\partial \mathbf{S}_c[p, k, \dot{\mathbf{a}}(p)]}{\partial a_{ij}} (a_{ij} - \dot{a}_{ij})$$

$$\mathbf{S}_s[p, k, \mathbf{a}(p)] = \mathbf{S}_s[p, k, \dot{\mathbf{a}}(p)] + \sum_{j=1}^J \sum_{i=1}^I \frac{\partial \mathbf{S}_s[p, k, \dot{\mathbf{a}}(p)]}{\partial a_{ij}} (a_{ij} - \dot{a}_{ij})$$

2. Vector-estimates-object intensities: $\dot{\mathbf{a}}(p) = [\dot{a}_{11} \dots \dot{a}_{1J}, \dot{a}_{21} \dots \dot{a}_{ij} \dots \dot{a}_{IJ}]^T$

3. Constant coefficients of Taylor expansion:

$$S_c[p, k, \dot{\mathbf{a}}(p)] = \sum_{j=1}^J \sum_{i=1}^I \dot{a}_{ij} \cos[\mathbf{w}(t - t_{ij}) + b(t - t_{ij})^2]$$

$$S_s[p, k, \dot{\mathbf{a}}(p)] = \sum_{j=1}^J \sum_{i=1}^I \dot{a}_{ij} \sin[\mathbf{w}(t - t_{ij}) + b(t - t_{ij})^2]$$

4. Linear coefficients of Taylor expansion:

$$\frac{\partial S_c[p, k, \dot{\mathbf{a}}(p)]}{\partial a_{ij}} = \cos[\mathbf{w}(t - t_{ij}) + b(t - t_{ij})^2]$$

$$\frac{\partial S_s[p, k, \dot{\mathbf{a}}(p)]}{\partial a_{ij}} = \sin[\mathbf{w}(t - t_{ij}) + b(t - t_{ij})^2]$$

5. State transition matrix function: $\mathbf{g}(p, k) = \text{diag} \left[\exp \left(\frac{NT_p}{t_{ij}} \right) \right]$

Kalman Matrix Equations

1. Recurrent estimation equation

$$\dot{\mathbf{a}}(p) = \mathbf{g}(p, k) \cdot \dot{\mathbf{a}}(p-1) + \mathbf{K}(p, k) \{ \hat{\mathbf{i}}(p, k) - \mathbf{S}[p, k, \mathbf{a}(p-1)] \}$$

2. New measurement vector

$$\hat{\mathbf{i}}(p, k) = [\hat{\mathbf{i}}_c(p, k), \hat{\mathbf{i}}_s(p, k)]^T$$

3. Measurement prediction vector

$$\mathbf{S}[p, k, \mathbf{a}(p-1)] = \begin{bmatrix} \mathbf{S}_c[p, k, \mathbf{a}(p-1)] \\ \mathbf{S}_s[p, k, \mathbf{a}(p-1)] \end{bmatrix}$$

4. Kalman filter gain

$$\mathbf{K}(p, k) = \mathbf{R}(p, k) \mathbf{H}^T(p, k) \mathbf{y}^{-1}(p, k)$$

5. Update state error covariance matrix

$$\begin{aligned} \mathbf{R}^{-1}(p, k) = & \mathbf{H}^T(p, k) \mathbf{y}^{-1}(p, k) \mathbf{H}(p, k) + \\ & + [\mathbf{g}^T(p, k) \mathbf{R}(p-1, k) \mathbf{g}(p, k) + \mathbf{V}^{-1}(p, k)]^{-1} \end{aligned}$$

6. Gradient matrix $\mathbf{H}(p, k)$

$$\mathbf{H}(p, k) = \begin{bmatrix} \frac{\partial \mathbf{S}_c[p, 1, \dot{\mathbf{a}}(p-1)]}{\partial a_{11}} & \dots & \frac{\partial \mathbf{S}_c[p, 1, \dot{\mathbf{a}}(p-1)]}{\partial a_{IJ}} \\ \frac{\partial \mathbf{S}_c[p, K, \dot{\mathbf{a}}(p-1)]}{\partial a_{ij}} & \dots & \frac{\partial \mathbf{S}_c[p, K, \dot{\mathbf{a}}(p-1)]}{\partial a_{IJ}} \\ \frac{\partial \mathbf{S}_s[p, 1, \dot{\mathbf{a}}(p-1)]}{\partial a_{11}} & \dots & \frac{\partial \mathbf{S}_s[p, 1, \dot{\mathbf{a}}(p-1)]}{\partial a_{IJ}} \\ \frac{\partial \mathbf{S}_s[p, K, \dot{\mathbf{a}}(p-1)]}{\partial a_{11}} & \dots & \frac{\partial \mathbf{S}_s[p, K, \dot{\mathbf{a}}(p-1)]}{\partial a_{IJ}} \end{bmatrix}$$

7. Elements of the gradient matrix $\mathbf{H}(p, k)$

$$h_{k, (i-1)J + j}^c = \frac{\partial \mathbf{S}_c[p, k, \dot{\mathbf{a}}(p-1)]}{\partial a_{ij}} = \cos[\mathbf{W}(t - t_{ij}) + (t - t_{ij})^2]$$

$$h_{k, (i-1)J + j}^s = \frac{\partial \mathbf{S}_s[p, k, \dot{\mathbf{a}}(p-1)]}{\partial a_{ij}} = \sin[\mathbf{W}(t - t_{ij}) + b(t - t_{ij})^2]$$

Numerical Experiment

1. Trajectory parameters: $V = 600 \text{ m/s}$; $\mathbf{a} = \mathbf{p}$; $j = 0$
 $x_{00}(0) = 0 \text{ m}$; $y_{00}(0) = 5 \cdot 10^4 \text{ m}$;
2. Parameters of transmitted pulses: $T_p = 2,5 \cdot 10^{-2} \text{ s}$; $I = 3 \cdot 10^{-2} \text{ m}$;
 $N = 100$; $\Delta T = 3,125 \cdot 10^{-8} \text{ s}$; $T = 10^{-6} \text{ s}$; $b = 9,425 \cdot 10^{14}$
 $K = 32$; $f = 10^{10} \text{ Hz}$; $\Delta F = 3 \cdot 10^8 \text{ Hz}$.
3. Parameters of geometrical model: $\Delta X = \Delta Y = 0,5 \text{ m}$;

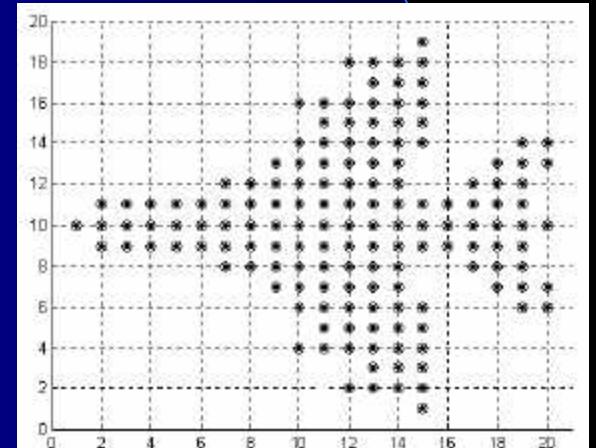
Number of point scatterers of 2-D grid $O'XY$

$$I = 20 \quad J = 20$$

Point intensities of the object: $a_{ij} = 0.01$

Point intensities out of the object: $a_{ij} = 0.001$

Initial intensities of the grid: $a_{ij} = 0.001$



Four phases of Kalman image reconstruction

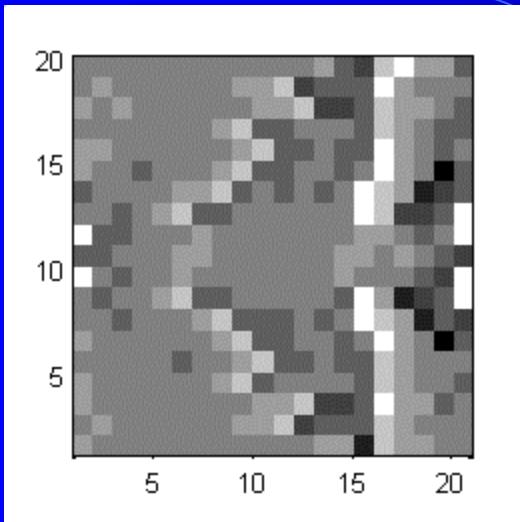


Fig 8. ISAR image by $p=40$ step.

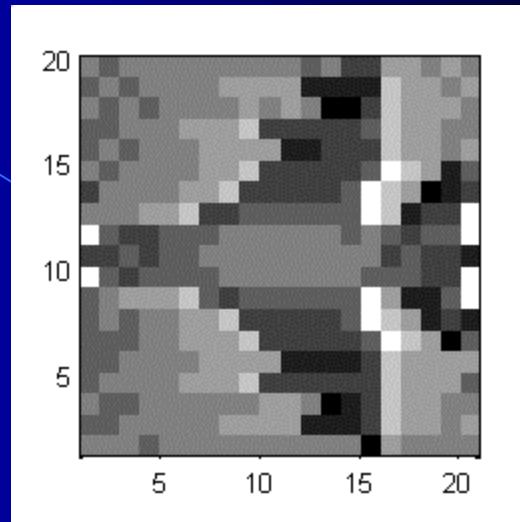


Fig 9. ISAR image by $p=45$ step.

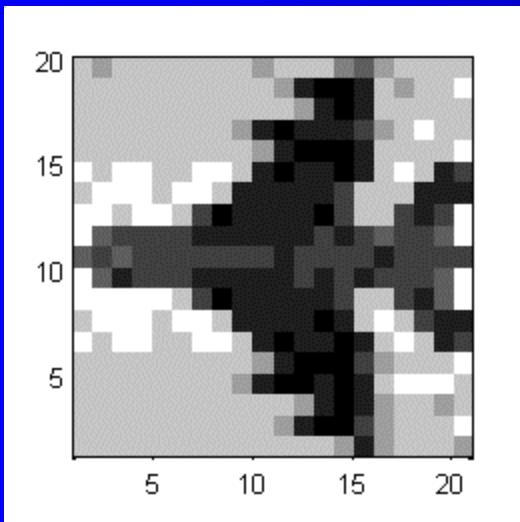


Fig 12. ISAR image by $p=75$ step.

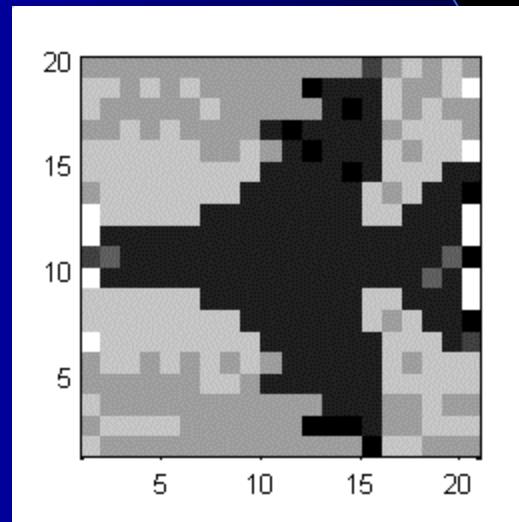
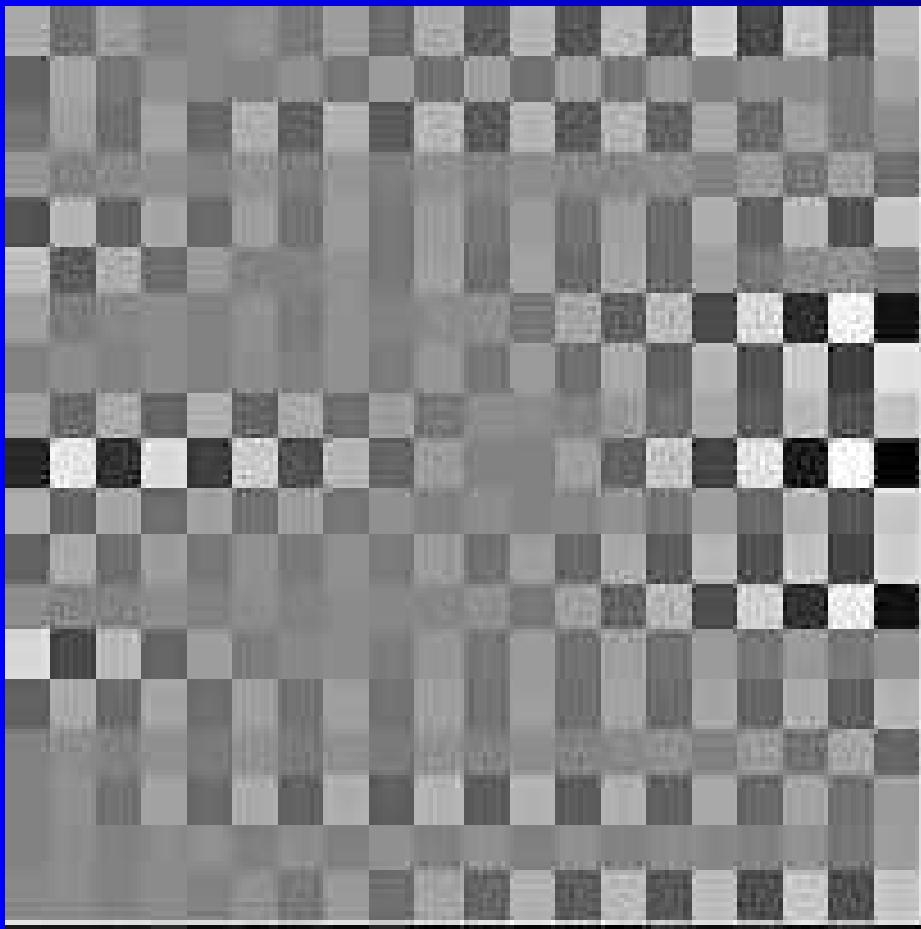


Fig 14. ISAR image by $p=100$ step.

Movie



Conclusion

1. The Kalman procedure extracts the target image by estimating the invariant geometric parameters (intensities) from quadrature components of the LFM ISAR signal.
2. The experimental results demonstrate that the Kalman method can be used to replace correlation and Fourier transforms as a means of ISAR image reconstruction. The recurrent procedure is characterized by high effectiveness and speedy convergence for the target image reconstruction using simulated ISAR data.
3. The results of the mathematical modeling LFM ISAR signals as well as developed Kalman algorithm for extracting geometric parameters from the trajectory complex LFM ISAR signal can be successfully used to develop a microprocessor system for the object's image restoration from ISAR data.